The computer as an aid for exploring graphs

■ Laurence T Rogers

Graphs have an important role in conveying scientific information, and the ability to read and use information from graphs and charts is now considered essential in science education. Pupils have more difficulties in using graphical techniques than teachers would like to suppose, especially in interpreting the meaning of graphs and relating physical variables. A new generation of computer software comes to the aid of pupils, not only providing opportunities for overcoming difficulties but also amplifying the value of graphs as tools for scientific investigation.

INTRODUCTION

Data-logging software makes especially valuable contributions to the quality and scope of practical work. It manages the collection, display, storage and analysis of data obtained from physical sensors and data-loggers connected to the computer. Allowing the computer to take on these roles has a profound effect on the skills required by pupils; it renders the manual skills involved in collecting, tabulating and plotting data redundant. For graphs, how much experience of manual plotting do pupils need? Might the traditional emphasis on manual skills impede the interpretation and understanding of graphs? What are the gains when the computer delivers the graph? To discuss these questions, the following observations will be considered for graphs produced by data-logging software.

The display of data can be *immediate*; the graph can be displayed while the data is being collected so that it is more easily associated with the phenomena it represents.

Qualitative display precedes *quantitative* analysis; this is less complex than the traditional process which requires the management of numerical data as a pre-requisite of the graphical display which reveals a pattern in the data.

Freed from the need to record the data item by item, pupils have more time to make careful *observations* of the phenomena in the experiment.

The analysis of the graph may become an 'interactive' process, encouraging pupils to test out their ideas about the data and seek its underlying meaning.

It will be argued that these attributes have great potential in shifting an emphasis in pupils' activity away from the *gathering* of data towards its *interpretation*. The computer's success in empowering pupils to explore the data partly depends upon the design of the software giving them confident control over suitable analysing aids. It also makes graphical presentation available to a wide range of users, including the less able, who might lack the skill or time to construct graphs by conventional means, but who nevertheless can take advantage of the insights which graphs can offer.

SKILLS REQUIRED FOR ANALYSING AND INTERPRETING GRAPHS

The work of Janvier has shown that, whereas most pupils may become proficient at the reading and plotting of graphs, the interpretation of graphs depends on the ability to understand global features such as intervals, maxima and minima, discontinuities and so on. Pupils are much less successful in these areas [1]. Janvier suggests that this could be due to the great emphasis placed by teachers on 'accurate' skills; choosing scales so that the graph will fit the paper, generating and plotting points, joining them up with a smooth curve, reading off isolated values etc. They appear to be emphasized to such an extent that the overall meaning of the graph and the significance of global graphical features are left unexplained in the pupils' minds. Swan suggests a teaching strategy to help correct the imbalance in this teaching through giving pupils practice in rough-sketching graphs to match given descriptions in words or pictures [2]. The activity encourages thinking about the connection between the features of a graph image and their meaning in terms of the physical events they represent. It has been suggested that the use of the computer also succeeds in shifting the emphasis in pupils' activity towards the interpretation of the data. To consider this, the particular skills employed in interpreting graphs will be surveyed.

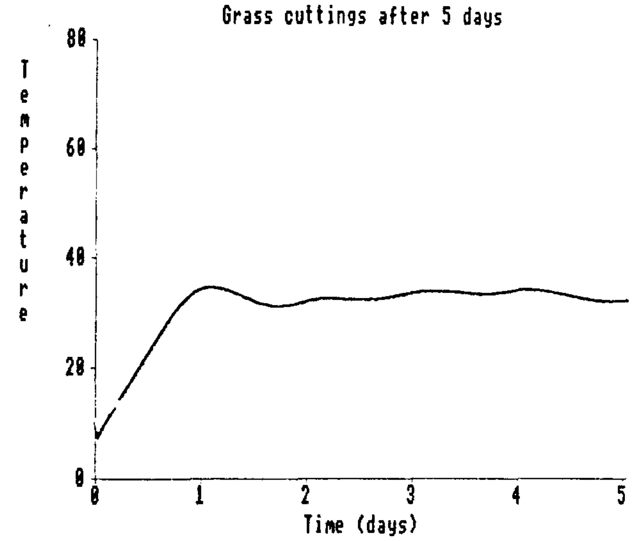
There are several levels of sophistication in the process of interpretation. At the simplest level, the graph shape may be viewed qualitatively, identifying trends and interesting features. Progressing to a quantitative treatment, information is obtained from the graph, reading values, performing simple calculations on coordinates and so on. Beyond this, the progression may involve attaching meaning, making generalizations and applying understanding derived from the graph. The following progression will be used in a discussion of interpretation skills:

Viewing graph qualitatively
Reading values
Describing variables
Relating variables
Making predictions
Translating descriptions into mathematical form

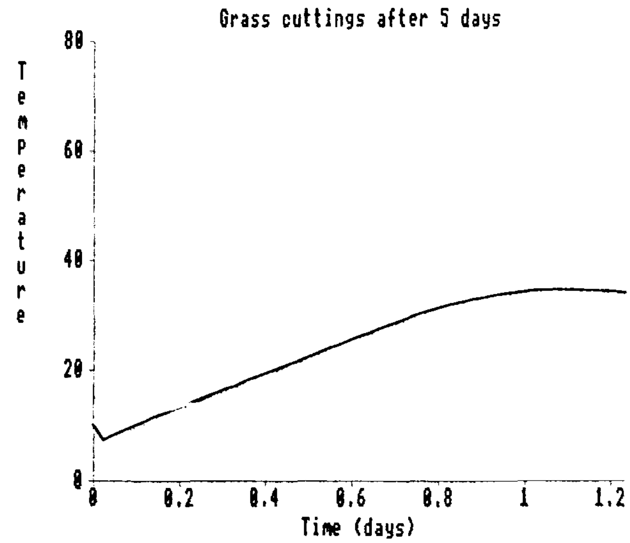
Viewing the graph

The most obvious feature of a graph is its shape. Potentially the shape can immediately convey information in a qualitative manner without concerning the observer with unnecessary, involved detail. The media and the press make prolific use of the graph as a device for communicating the 'feel' for a trend or a relationship. Likewise in science, without any recourse to numerical or quantitative consideration, pupils can gain a glimpse or a quick overview of what may be going on in an experiment; they can 'see' gradual or sudden changes, continuity or discontinuity and can select and give attention to particular interesting features. It is a valuable characteristic of data-logging software that the graph can be built up as the experiment proceeds which makes it more easily associated with the phenomena it represents. Also, this visual representation of the data is the first to be presented to pupils; the numerical attributes can follow when needed. Thus the traditional role of quantitative representation (tabulated results) being the pre-requisite of the qualitative image (graph) can be reversed, allowing pupils to explore the data qualitatively at first.

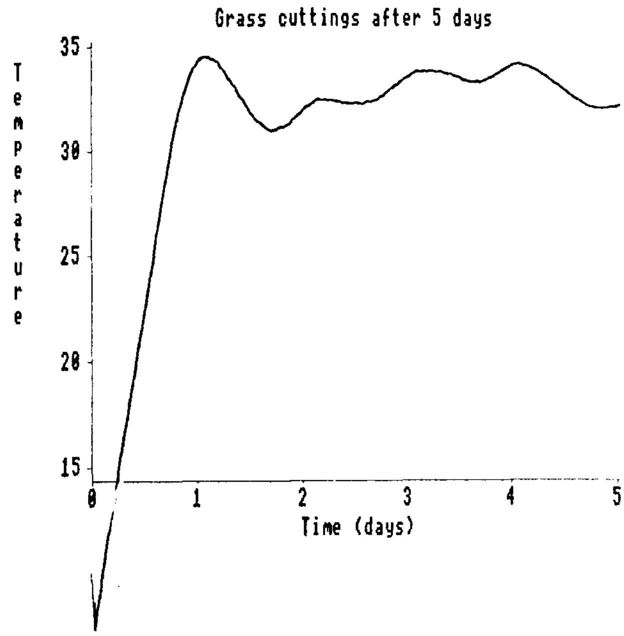
A generally weak aspect of graphwork tends to be in pupils' understanding and use of scale markings. Errors frequently occur when a coordinate does not coincide with a scale marking or grid line, when more judgement is needed for placing a point. The effect of scales on the shape of a graph is a further aspect which reveals weak understanding. Unfortunately the time needed to plot a graph manually does not encourage the pupil to repeat the exercise if an unsuitable choice of scale is made. This is a clear case where the computer can come to the aid of the pupil and offer distinct advantages. Not only is the plotting accurate, but the speed of plotting enables graphs to be treated dynamically without delay, encouraging them to experiment with scales etc and generally explore their data.



(a) Full scale view



(c) Horizontal scale expanded



(b) Vertical scale expanded

Figure 1 The apparatus dependence of the magnitudes and rates of change on the vertical and horizontal plotting scales may be readily explored with the zoom facility. Note how the software automatically calibrates the scales with 'friendly' numbers

For example, the appearance of the rate of change of a variable depends upon the choice of scales. The zoom facility in software is a particularly useful tool for adjusting the magnification and scales to provide a variety of different views of the data (Figure 1).

Interpretation of graph shape does not necessarily require axes to be calibrated, but clearly this imposes a limit to the precision of description. Descriptions in simple words can provide valuable stimuli to pupils' thinking and understanding, but the need for more precise description demands a more quantitative approach. Surveys have shown that, in investigative work, children tend to opt for qualitative approaches to practical work and are disinclined to adopt a more quantitative approach, because of the perceived increase in the complexity of the experiment [3, p 98]. This reluctance has been identified in pupils even when they have demonstrated competence in constructing graphs in prescribed contexts [4, p 21]. Here, software can support the pupils by reporting initially in a qualitative mode, but then helps to develop their ideas about the value of quantitative evidence through a variety of analysing and calculating aids.

Reading values

Surveys have revealed that pupils tend to find difficulty in reading analogue scales correctly [5]. The interpolation of values between scale markings, and an understanding of subdivisions and decimal places are typical casualties. The problem appears to be compounded when two or more readings need to be compared and used to calculate quantities such as gradients and intervals. Using software, reading and calculating data from graphs becomes an almost trivial activity, merely requiring the pupil to manoeuvre a pair of cross hairs to read off values which are displayed in a panel on the screen. This type of facility may be used to gain automatic readouts of coordinates, time intervals, differences, ratios, gradients, areas, mean values, maxima and minima.

In the traditional context of graphing skills it may seem an abuse to deny pupils opportunities to perform these tasks manually. However, by reducing these skills to a low level, higher order skills may flourish, empowering pupils to think more about the science of their experiment. This has an important impact on investigative and problem-solving approaches to practical science where the problem-solving process relies on service skills such as measurement not requiring thought; they should be effortless and as automatic as possible. The lack of automism interrupts the problem-solving process and can lead to errors [6, p 30].

Typically, the reliability of derived data generated by computer calculations is considerably greater than that which depend on pupils' own arithmetic. Given that this enhancement makes the pupil better informed, we might hope that their judgements based on the information might also be better.

Time dependent data are some of the most common in pupils' experience of school science. As a variable, time has a special quality; pupils have a natural 'feel' for it through a sequence of events or changes in other variables. Sometimes, so strong is the feeling of the schedule of events, pupils tend to interpret data in a 'pointwise' fashion rather than the intervals between them [7, p 37]. Software can compensate for this and help strengthen the notion of intervals of time by allowing pupils to choose an arbitrary origin for reading of time measurements. It can also exclude errors in reading the correct number of decimal places for the interval measurement which often occurs when intervals are less than the interval between consecutive scale markings on the axes. This is similar to the difficulty which pupils experience in interpolating values between the scale markings. Both of these problems are alleviated when the software automatically adjusts the spacing of the scale markings according to the magnification and uses 'friendly' numbers for the scale markings so that interpolation only requires estimates of simple fractions of the scale interval.

Discrete data points on a graph can focus too much attention to the points such that terms like 'maxima' and 'minima' tend to be confused with the actual height on a graph rather than being identified with changes in the steepness of the graph. Using the computer, data logging typically involves much larger amounts of collected data plotted more densely on the graph, which helps to disguise the discreteness of the data and emphasize rate of change. Analysing aids make steepness (or gradient) easily computed and display features can reinforce the concept of rate of change. For example, horizontal and vertical cursor lines may be locked on to the data so that as the pupil controls the movement of one cursor, the other cursor is constrained to follow the data values. The resulting relative movement of the two cursors gives a dynamic and visual indication of the rate at which one variable changes with respect to the other (Figure 2).

The confusion between slope and height might alternatively be an example of lin-

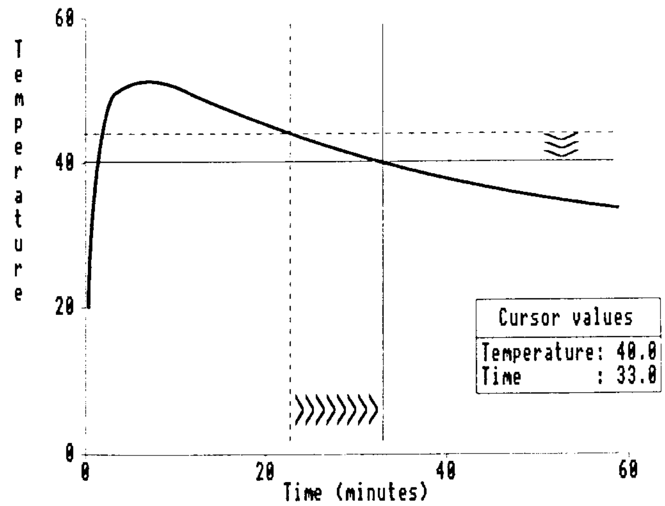


Figure 2 As the vertical cursor is dragged across the graph, the corresponding movement of the horizontal cursor is visibly faster or slower according to the gradient of the data line. The software constrains the intersection *f* the cursors to lock on to the data line

guistic problem in which pupils' tend to interpret any words of magnitude casually as 'big' or 'small' without a clear context of the property being described. To make matters even more complex, this might also be compounded with conceptual difficulties associated with variables. A common example is the confusion between velocity and acceleration which can cause the words to be used synonymously. Here software can assist by providing opportunities for working interactively with the data; through a range of display, analysing and calculating aids, pupils can readily manipulate their data to probe and test their understanding.

Describing variables

To describe a variable, pupils need both to obtain information from a graph and also attach meaning to the information. It was noted above that qualitative descriptions may be based on the shape of a graph without any explicit reading of data values. To progress to a quantitative treatment, at least two items of information need to be taken from a graph and then compared in some way. Software analysing aids provide useful assistance in making the types of measurement which benefit such descriptions:

Maximum, minimum and mean values
Difference or ratio between two values
Gradient for rate of change in a variable

The quantity of data contained in a computer graph naturally lends itself to the study of connections, patterns and trends in the data. The visual aspect of the graph draws attention to these features more effectively than numbers in columns of tabulated values. Indeed, there is evidence that pupils are distracted from making generalized descriptions by obvious numerical patterns in tabulated results and instead described patterns such as sequences of odd and even numbers, multiples and differences etc [8, p 28]. Numerical patterns are more difficult to spot when the data is not in serial order but for manual measurements serial data is common since most pupils are trained to increment or decrement the independent variable in an orderly manner. It is unfortunate that tabulated data encourages this type of stepwise analysis which misses the continuity in the behaviour of a variable. Even with a graph, when asked to draw a line through their data, pupils often attempt to join successive points with straight lines, again illustrating this stepwise perception. Software can help to move pupils' thinking towards a continuous view of the data by plotting a best-fit curve which emphasizes an underlying trend represented by the shape of the curve. However, even when pupils focus appropriate attention to the shape of the graph, this does not necessarily result in descriptions of a variable; some pupils instead describe the geometry of the graph line in terms of its shape, direction or curvature in isolation from the axes and from what those axes represent [8, p 29]. A similar geometrical viewpoint is revealed when pupils interpret a graph as the actual picture of situation; for example, going up and down hills in distance-time graphs [7, p 39].

Much of this evidence suggests that the process of obtaining scientific descriptions of variables is fraught with distractions; pupils' descriptions are offered at a number of different levels of sophistication. One approach to deflecting attention from numbers, points and geometrical properties is to use software tools for manipulating the data, providing a variety of alternative views and presentations of the data. For example the data may be smoothed to reduce the effect of 'noise' and emphasize the trend; short-term and longer-term changes may be distinguished; variations in the trend may be observed; the data may be overlaid and compared with a standard mathematical curve; pupils may experiment with matching 'trial fit' curves to their data; differences or 'residuals' between the fitted curve and the data may be plotted [9, p 10]; scatter graphs indicate a correlation between two variables; a first derivative (Gradient) curve may be plotted. Through a variety of representations, pupils may be encouraged to think about the physical variable rather than the numbers and images which represent it. Software provides quick and convenient tools for all of these manipulations [10, p 7]. In addition, datalogging in 'real time' helps to forge a link between the physical variable and its graphical representation since the latter appears on the screen simultaneously with the collection of data. The promptness of display makes an interactive experience possible whereby pupils can alter conditions in an experiment and immediately observe a response [11].

Relating variables

Identifying and describing a trend or pattern in the behaviour of a variable marks an important stage of sophistication in interpreting graphs because it indicates a perception which is generalized beyond the actual items of data presented on the graph. The majority of graphs generated by datalogging software typically show the time dependence of one or more variables, so the description of a pattern implicitly relates the physical variables to the time variable. Such descriptions are very frequently concerned with changes, rates of change, growth and decay, which are compound variables relating a primary variable with time.

Describing the *relationship* between variables in a generalized manner is the key to developing scientific ideas from the graphs, but pupils often find this difficult [12, p 15]. Sometimes their difficulties are linguistic; they are unsure of the type of expression required; as previously indicated, they may find it easier to use geometrical or numerical descriptions rather than referring to the physical variables. Their terminology may lack precision; the use of vague terms such as 'goes up' instead of 'increases rapidly' might imply that they are perceiving variables separately and are not consciously relating them. Linguistic considerations may disguise pupils' understanding, but even withstanding this, it is a big step for pupils to see the graph as showing the relationship between two variables [13]. Pupils need a teaching strategy which helps them ask appropriate questions and which nurtures and gives them practice in appropriate skills. They also need suitable tools for exploring graphs, tools which are readily provided in software.

The shape of a graph gives vital information about the relationship between the variables. A straight line or a curve have distinctive properties which provide insight and understanding of that relationship.

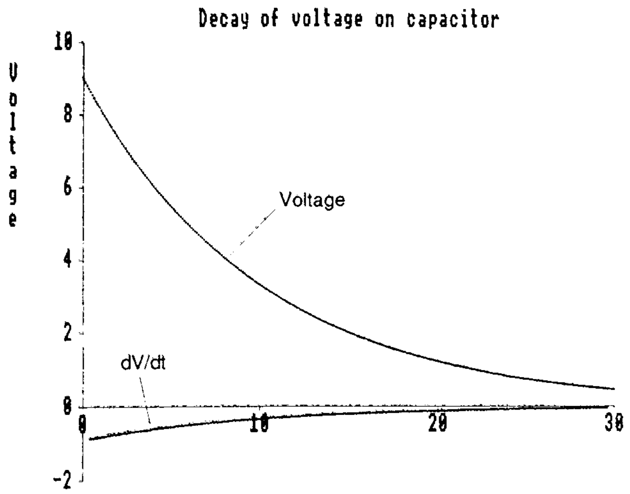
For a relationship represented by a straight line, a variety of statements may be made about its properties.

1 Changes in the variables occur at a constant rate.
2 For a given increment in one variable, the other variable always increases or decreases in equal steps.
(For the case of a variable plotted against time, the size of the step for a given time interval is always the same.)
3 This is independent of the magnitude of either variable.
4 The ratio between the increases or decreases in either variables is constant.
5 When this ratio is not unity, one variable changes more rapidly than the other.
6 The gradient is the same at all places on the graph, ie, it is constant.
7 When the gradient is negative, an increase in one variable is accompanied by a decrease in the other.

To the tutored eye, these descriptions are clearly equivalent to or follow from each other. Their significance here is that they are individually testable using software tools: when a cursor is moved across the graph, changes in the variables may be read automatically and the rate of change calculated; measurements may be taken from any selected part of the graph; '*x*' and '*y*' cursors may be locked together, easily showing the relative changes in two variables; the gradient at a cursor may be read automatically. It can be seen that this provides pupils with a variety of ways of exploring the properties of a linear relationship. The computer tools make these explorations viable and worthwhile, broadening pupils' experience of these properties.

The same tools and methods may also be used to explore relationships represented by curves:

1 One variable changes more rapidly than the other.
2 The rate of change varies across different parts of the graph.
3 The degree of curvature shows how rapidly the rate varies.
4 Exponential behaviour may be identified when the rate of change varies in direct proportion to the vertical variable (Figure 3).



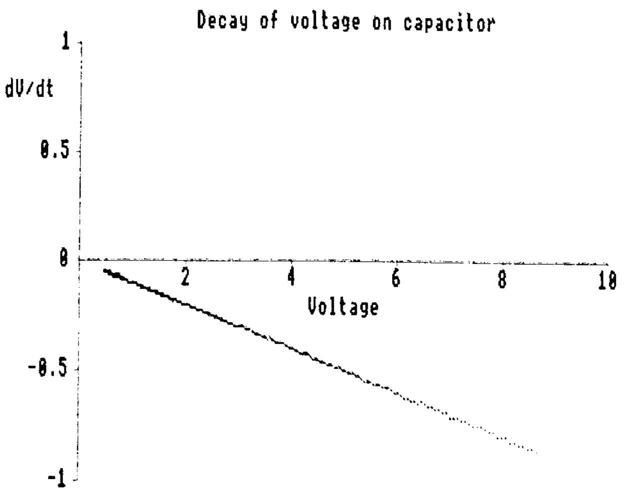


Figure 3 After logging the voltage decay on a capacitor, software may be used to calculate and plot the rate of decay *dV/dt*. In turn *dV/dt* may be plotted against *V* to test for proportionality, a property of exponential decay

Making predictions

When a pupil has succeeded in elucidating a generalized relationship between two variables, it will not necessarily indicate that one physically influences the other, but it should make possible interpolation and extrapolation to predict new data values. Thus the successful interpretation can be put to the test by using it to predict new values. Software tools such as best fit curves support pupils in developing confidence in the concept that the description transcends the items of collected data and enables them to identify values between and beyond these items.

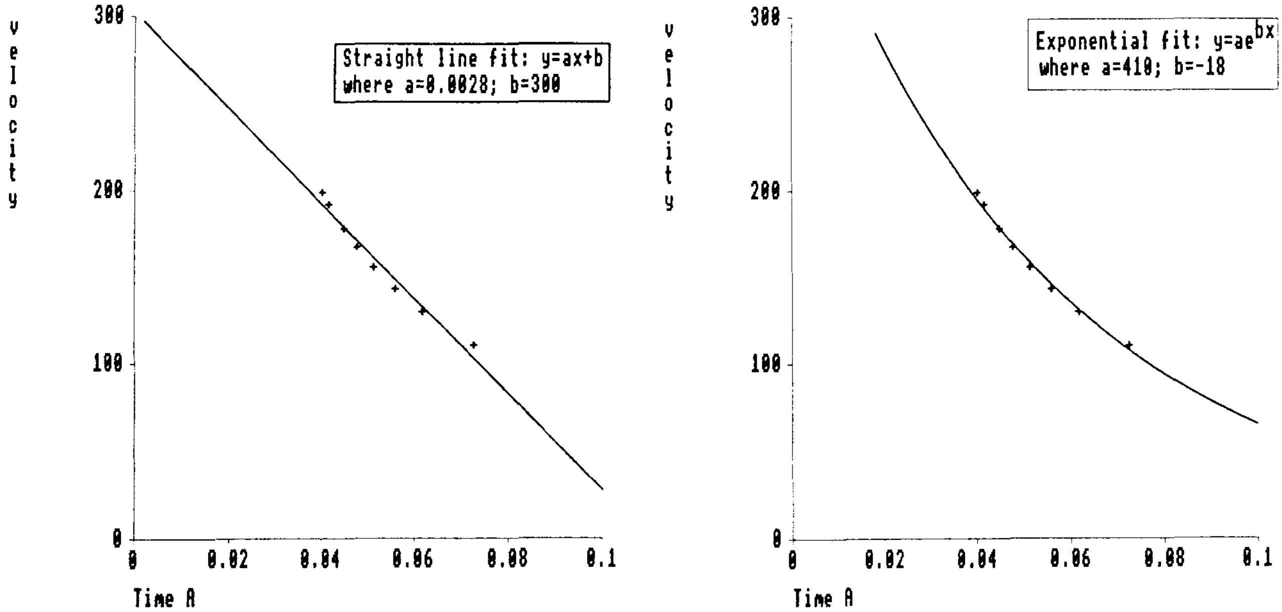
Similarly, pupils might predict data and description of the graph shape for new compound variables. For example, predicting a velocity-time graph from a distance time graph, or a power-voltage graph from a current-voltage graph. Software provides a variety of convenient calculating facilities for generating new data from the collected data allowing pupils to test their prediction.

Translating description into mathematical forms

A further stage of refinement of interpreting skill involves translating the relationship into an algebraic representation. Trends may be classified and described more precisely by association with a mathematical function, but unless the pattern is linear, the manual method of identifying the appropriate function usually depends upon transforming one variable and replotting it in the hope of obtaining a linear graph. This process can be convoluted, but software offers a number of alternative strategies:

1 The data may be linearized by a suitable function chosen by the pupil.
2 Pupils may simulate a function, compare it with the graph of the data and alter the parameters of the function until the best match is obtained.
3 Curve-fitting techniques.

Software curve-fitting facilities generates very attractive smooth curves, but often their mathematical description consists of complex polynomials expressions. In a version called 'Trial Fit' [10] the pupil can choose a simple general form of mathematical curve and experiment to find out the quality of the fit. The tool is designed so



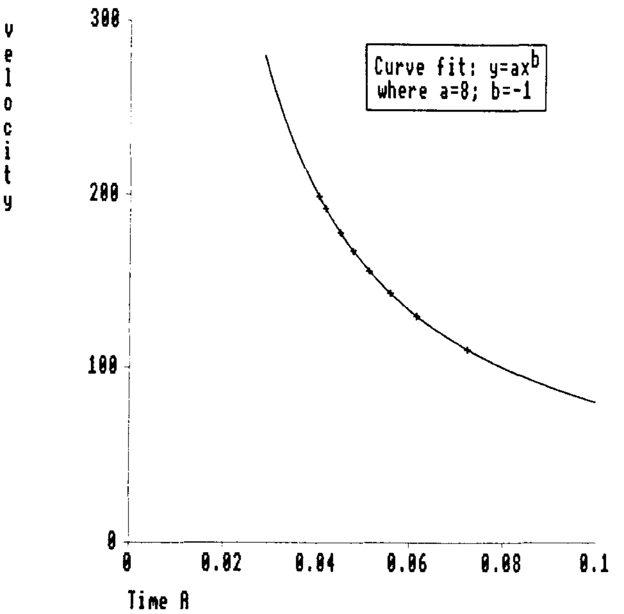


Figure 4 'Trial Fit' curve fitting: pupils may experiment to find out which type of curve most appropriately describes the experimental data

that the pupil is allowed to make a judgement of this quality. The virtue of restricting the choice to three simple forms is that pupils might readily associate them with simple numerical descriptions. The three forms available have been chosen to identify the most common relationships found in scientific data:

Linear Used for identifying proportionality, and extrapolating to find offsets and starting values.

Power Law curves Used for identifying parabolic, inverse and inverse square relationships.

Exponentials Used for identifying exponential growth or decay.

As in previous examples, the speed of calculation and plotting of data through software provides an interactive tool for pupils. Different types of fit may be tried in rapid succession, and curves may be compared by overlaying so that pupils can look for similarities and differences (Figure 4).

SOFTWARE DESIGN

Effectively designed software gives pupils a large measure of autonomy over the process of collecting and analysing data. The systems offered for control and the appearances of the screen have crucial roles in helping pupils select and exploit what features will be used. If the screen is cluttered with an excess of information, there is a risk that the pupil will be distracted from the purpose of the graph analysis. The 'tool' approach (content-independent) to software is successful when it allows the user to specify which features are to be enabled or hidden and this minimizes the number of mandatory decisions when the program is used. However, it is important to give pupils control at a level they can cope with. This requires configuring default conditions so that they can get started easily and build their confidence.

TEACHING AND LEARNING STRATEGIES

This article has surveyed a range of pupils' difficulties in using graphical techniques and has argued that graphing software and data-logging software offer significant benefits towards helping pupils overcome these difficulties. However, these software tools and the skills to use them are not enough on their own. Misconceptions cannot be removed by mere 'exposure' to the information contained in graphs. The graph is a thinking aid, but pupils need time to practise describing and using patterns to engage in the necessary reflection upon their results and discussion with their peers and teacher [14, p 42; 12, p 16]. Fortunately, the computer is good at providing time because it performs its tasks so rapidly. It frees pupils to devote more attention to observation, reflection and discussion [15].

It is also necessary for teachers to challenge pupils with key questions which encourage the type of thinking that goes beyond the simple reading of information from the graph and description of a variable, and leads to an interpretation in terms of a generalized relationship between variables. The graph should be considered as a resource and starting point for thinking activity rather than an end point of lesson activity. It also requires vision of methods for exploiting software; methods for comparing data, using cursors, performing calculations, fitting curves, altering scales etc.

Finally, teachers need to review the relative value they attach to different aspects of graphical technique. Pupils' success or failure depends on the incentives to meet targets set by the teacher. Targets implicitly indicate what is valued and important. If the emphasis is to be effectively shifted away from the routine collection and plotting of data towards the development and use of interpreting skills, the system of rewards and assessment of pupil performance needs to reflect this.

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LT Rogers, School of Education, University of Leicester, 21 University Rd, Leicester LE1 7RF